



# General Ether Theory and Graviton Mass

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## Abstract

For negative cosmological constant  $\Lambda$  we show the equivalence of the Lagrangian of “general ether theory” with Logunov’s “relativistic theory of gravity” with massive graviton and a variant of GR with four non-standard scalar fields.

We consider the remaining differences between these theories.

## 1 Introduction

In [13], a metric theory of gravity in a predefined Newtonian framework with Galilean coordinates  $T(x), X^i(x)$  with Lagrange density

$$L = (R - \Lambda)\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) + \Xi g^{\mu\nu} \delta_{ij} X_{,\mu}^i X_{,\nu}^j \sqrt{-g} - \Upsilon g^{\mu\nu} T_{,\mu} T_{,\nu} \sqrt{-g}$$

This theory allows a simple condensed matter interpretation. This condensed matter (“ether”) interpretation may be used to derive the Lagrange density. In this derivation, the constants  $\Xi$ ,  $\Upsilon$  and Einstein’s cosmological constant  $\Lambda$  remain unspecified, even their signs. We add here another simple hypothesis which allows to specify the signs: we assume that there exists an “undistorted reference state” – a solution with constant  $\rho, v^i, \sigma^{ij}$  – and that this undistorted reference state is stable. This hypothesis fixes the signs as  $\Xi > 0$ ,  $\Upsilon > 0$ . Moreover, it requires  $\Lambda < 0$  for Einstein’s cosmological constant.

With these sign conventions, the Lagrange density may be transformed in the preferred coordinates into

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$$L = R\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) - m_g^2(\frac{1}{2}\eta_{\mu\nu}g^{\mu\nu} - 1)\sqrt{-g}$$

where  $\eta_{\mu\nu}$  defines the vacuum solution and  $m_g$  the mass of the graviton in the vacuum state. This was an unexpected result – the Lagrangian looks like the usual GR Lagrangian with some additional scalar fields  $X^i(x)$ ,  $T(x)$ , and for  $\Xi > 0$ ,  $\Upsilon > 0$  they simply lead to additional “dark matter” terms with pressure  $p = -\frac{1}{3}\varepsilon$  resp.  $p = \varepsilon$ . Therefore, I have assumed that the graviton remains mass-less.

After this, it was reasonable to compare the theory with existing theories with massive graviton. This search has been successful, we have found a theory with the same Lagrangian – “relativistic theory of gravity” developed by Logunov a.o. [9].

Thus, the Lagrangian has been derived independently based on completely different motivation. This is not strange, because the harmonic condition – the simplest and most beautiful coordinate condition – is used in above theories and is all what is necessary to obtain the Lagrange formalism.

Some interesting properties of the theory in the limit of very small  $m_g \rightarrow 0$  are easy to understand – we obtain an oscillating universe, stable “frozen stars” of  $\gtrsim$  Schwarzschild size, a bounce for the gravitational collapse. Thus, it is a nice regularization of GR. Even for arbitrary small  $m_g$  this solves cosmological problems – the horizon problem and the flatness problem – solved in standard cosmology with inflation (cf. [12], p.5,56). Thus, we do not have to introduce inflation to solve these problems.

Above theories have the same Lagrangian, but there are not only major differences in the metaphysical interpretation, but also minor but interesting differences in predictive power and differences in the quantization concepts related with these theories.

## 2 General Ether Theory

The basic formula is the definition of the physical metric  $g^{\mu\nu}$  as a function of typical condensed matter variables (density  $\rho$ , velocity  $v^i$ , stress tensor  $\sigma^{ij}$ ):

$$\hat{g}^{00} = g^{00}\sqrt{-g} = \rho \tag{1}$$

$$\hat{g}^{i0} = g^{i0} \sqrt{-g} = \rho v^i \quad (2)$$

$$\hat{g}^{ij} = g^{ij} \sqrt{-g} = \rho v^i v^j - \sigma^{ij} \quad (3)$$

This matter (the “ether”) fulfils classical conservation laws:

$$\partial_t \rho + \partial_i (\rho v^i) = 0 \quad (4)$$

$$\partial_t (\rho v^j) + \partial_i (\rho v^i v^j - \sigma^{ij}) = 0 \quad (5)$$

Additional “inner steps of freedom”  $\psi^m(x)$  are also allowed, but no other, external matter. Thus, there are no momentum exchange terms for interaction with other matter. The “inner steps of freedom” of the “ether” are identified with usual matter fields.

In the metric variables, the conservation laws transform into the harmonic equation for the Galilean coordinates:

$$\square X^i = \square T = 0$$

If we search for a Lagrange density  $L(g^{\mu\nu}, \psi^m, X^i, T)$  which leads to these equations, we obtain the Lagrangian

$$L = L_{GR}(g_{\mu\nu}, \psi^m) + \Xi g^{\mu\nu} \delta_{ij} X^i_{,\mu} X^j_{,\nu} \sqrt{-g} - \Upsilon g^{\mu\nu} T_{,\mu} T_{,\nu} \sqrt{-g}$$

with unknown constants  $\Xi, \Upsilon$  almost immediately: the simplest way to obtain the harmonic equation for a field is the standard scalar Lagrangian plus the requirement that the remaining part does not depend on this field. But the requirement that the remaining part does not depend on the preferred Galilean coordinates is the requirement for the Lagrangian of GR.

This Lagrange formalism and the choice of independent variables seems strange from point of view of classical condensed matter theory. It defines a promising analogy between condensed matter theory and fundamental particle theory which is far away from being completely understood.

In this derivation, the signs of the cosmological constants remain unspecified. They should be defined by observation. Current observation of the “dark matter” seems to favour  $\Xi > 0, \Lambda > 0$ . For  $\Upsilon > 0$  we obtain interesting new effects – especially, we can solve the cosmological horizon problem without introducing inflation theory. Therefore, we tend to favour  $\Upsilon > 0, \Xi > 0, \Lambda > 0$ .

## 2.1 GET with negative cosmological constant

Nonetheless, another sign convention seems very interesting from theoretical point of view:  $\Upsilon > 0, \Xi > 0, \Lambda < 0$ . In this case, there exists a constant stable “vacuum solution”. Indeed, if there is no matter, the equation for the constant solution is:

$$\begin{aligned} ds^2 &= a^2 dt^2 - b^2(dx^2 + dy^2 + dz^2) \\ G_0^0 = 0 &= -\Upsilon a^{-2} + 3\Xi b^{-2} + \Lambda \\ G_1^1 = 0 &= +\Upsilon a^{-2} + \Xi b^{-2} + \Lambda \end{aligned}$$

with  $\Lambda = -2\Xi b^{-2} = -2\Upsilon a^{-2}$  as the only solution. In this situation, it seems natural to renormalize the constants and to introduce the vacuum state as  $\eta^{\mu\nu}$  into the Lagrange density.

$$L = R\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) + \Lambda(\frac{1}{2}\eta_{\alpha\beta}g^{\mu\nu}X_{,\mu}^\alpha X_{,\nu}^\beta - 1)\sqrt{-g}$$

It is stable if we choose  $\Lambda < 0$ . Indeed, let's consider the linearized equations for a small modification of the undistorted state  $g^{\mu\nu}(x) = \eta^{\mu\nu} + h^{\mu\nu}(x)$ . We obtain

$$\frac{1}{2}g^{kl}\frac{\partial h^{ij}}{\partial x^k\partial x^l} = -\frac{\Lambda}{2}h^{ij} + T^{ij} - \frac{1}{2}g^{ij}T$$

Thus, the theory becomes a theory with massive graviton with mass  $m_g = \sqrt{-\Lambda}$ . We obtain:

$$L = R\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) - m_g^2(\frac{1}{2}\eta_{\alpha\beta}g^{\mu\nu}X_{,\mu}^\alpha X_{,\nu}^\beta - 1)\sqrt{-g}$$

In the preferred coordinates, this Lagrange density is

$$L = R\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) - m_g^2(\frac{1}{2}\eta_{\mu\nu}g^{\mu\nu} - 1)\sqrt{-g}$$

## 3 Comparison of GET with similar theories

The Lagrange density for GET if  $\Xi > 0, \Upsilon > 0, \Lambda < 0$  is equivalent to the Lagrange density in [11], formulas (9),(10), for the “relativistic theory of gravity” (RTG) with non-zero graviton mass. On the other hand, if  $\Xi > 0, \Upsilon < 0$

we can formally obtain a similar Lagrangian if we introduce “clock fields”  $X^\mu(x)$  as scalar fields into GR (Kuchar [8] has considered such theories). Thus, this Lagrangian occurs with different motivation in three theories with completely different metaphysics. Let’s introduce the following notions:

- ADM – a variant of general relativity with dark matter fields  $X^\mu(x)$  (no background):

$$L = (R - \Lambda)\sqrt{-g} + L_{matter}(g_{\mu\nu}, \psi^m) - \Xi g^{\mu\nu} \eta_{\alpha\beta} X_{,\mu}^\alpha X_{,\nu}^\beta$$

- RTG – Logunov’s relativistic theory of gravity with massive graviton (Minkowski background):

$$L = L_{Rosen} + L_{matter}(g_{\mu\nu}, \psi^m) - m_g^2 \left( \frac{1}{2} \eta_{\mu\nu} g^{\mu\nu} \sqrt{-g} - \sqrt{-g} - \sqrt{-\eta} \right)$$

There is an additional causality condition: the light cone of  $g_{\mu\nu}$  should be inside the light cone of  $\eta_{\mu\nu}$ .

- GET – the generalization of Lorentz ether theory to gravity proposed by the author, for  $\Upsilon > 0, \Xi > 0, \Lambda < 0$  (Newtonian background):

$$L = L_{GR} + \Xi g^{\mu\nu} \delta_{ij} X_{,\mu}^i X_{,\nu}^j \sqrt{-g} - \Upsilon g^{\mu\nu} T_{,\mu} T_{,\nu} \sqrt{-g}$$

The causality condition in this theory is  $g^{00} \sqrt{-g} > 0$ .

Now, the common Lagrangian leads to common predictions which distinguish these three theories from classical GR: stable “frozen stars” near Schwarzschild size instead of black holes [10],[13], with bounce after gravitational collapse [10], a big bounce instead of a big bang singularity [9],[13] with an oscillating universe [9],[11]. Note that in an oscillating universe there is no horizon problem, and we have a natural preference for zero curvature. That means, two of the problems used to justify inflation theory (cf. [12]) disappear.

These seem to be common effects of theories with massive graviton, another way of introducing mass considered by Visser leads to similar results about the behaviour near the horizon [14]. Even in the limit  $m_g \approx 0$  the

qualitative differences remain. Note that once  $\Upsilon > 0$  ADM contains “matter” which violates all energy conditions. That’s why GR theorems about black hole and big bang singularities do not apply.

Let’s now consider the differences. First, we have a simple relation: A solution of GET or RTG defines a solution of ADM. In the other direction, this is not correct. The fields  $X^\mu(x)$  of a solution of ADM may not define a system of coordinates. And the solution of RTG possibly violates the condition  $\rho(x) > 0$ . That means, if GET resp. RTG are true, ADM cannot be falsified. But observing a solution of ADM where the four fields do not define global coordinates – for example, a solution with nontrivial topology – falsifies RTG and GET without falsifying ADM. In this sense, RTG and GET have higher predictive power.

To compare GET and RTG, the causality condition seems to be important. The causality condition of RTG is stronger than the causality condition of GET. Therefore, there may be GET-solutions which violate RTG causality. In this sense, the predictive power of RTG is higher. In GET, there is also no restriction for the sign of  $\Lambda, \Xi, \Upsilon$ , while RTG fixes these signs uniquely.

An interesting question about the physical meaning of the causality conditions in RTG and GET is if there are solutions which do not violate causality conditions for the initial values and what is their physical meaning. The answer for GET is that for solutions there  $\rho$  becomes zero the “continuous large scale approximation” described by GET fails, and we observe new physics – atomic ether theory.

Note that there are other important differences between GET and RTG. GET is compatible with the EPR criterion of reality [4] and with realistic, deterministic, but non-local hidden variable theories for QM like Bohmian mechanics [3]. Therefore, despite the fact that the three theories have equivalent Lagrangian formalism, and therefore equivalent field equations, it is worth to distinguish them as different physical theories.

These differences between the theories suggest essentially different concepts for quantization. In ADM we have the full beauty of canonical GR quantization problems, especially the problem of time [6] and topological foam. The harmonic condition may be used, but is only a gauge condition [8]. Instead, in GET and RTG it is a physical equation. The conceptual problems of GR quantization related with the absense of an absolute background are not present.

In GET quantization we can use condensed matter analogies (see for example Volovik [15]) as simple guiding principles. This suggests to use some “atomic ether” hypothesis which leads to an explicit, physical regularization. The “ether hypothesis” allows to obtain a prediction about the cutoff length:  $\rho(x)V_{crit} = 1$  in appropriate units. This prediction violates the relativistic invariance of the Lagrangian and is therefore incompatible with RTG symmetry. It differs also from Planck length suggested by the “Planck ether” concept [7], [15].

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